VI. When $x = 143.27$ then $y < 0$ and $A = 0$. Since $x < 192.5$, $(2x + y)2x > 0$ by (5). And since $A = 0$, $(2x + y)2x = |70y|$. Fig. 12 is again representative and the positive and negative areas just balance.

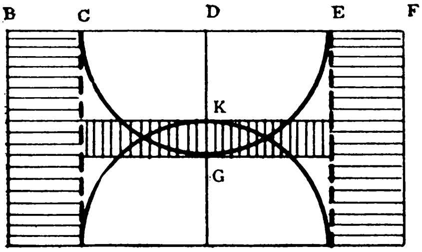

FIG. 12.
$102.67 < x < 192.5, y < -x.$

VII. When $x > 143.27$ then $y < 0$ and $A < 0$. On account of (5) we must consider three cases.

(a) When $143.27 < x < 192.5$, $(2x + y)2x > 0$ by (5). And since $A < 0$, $(2x + y)2x < |70y|$, *i. e.*, the area of the rectangle $GK \times CE$ is less than the area of the grandstands. Fig. 12 is again representative.

(b) When $x = 192.5$, $(2x + y)2x = 0$ by (5) and $A = 70y$, a negative area. Fig. 13 represents the tract.

(c) When $x > 192.5$, $(2x + y)2x < 0$ by (5). Now all of the areas included in A are negative and Fig. 14 is representative.

In V, VI, VII, the race-track is partly positive and partly negative. In equation (2) the $2\pi x$ is positive and the $2y$ is negative as is indicated in the figures by the solid and dotted parts of the track.

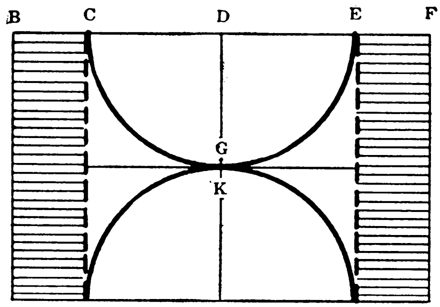

FIG. 13.
$x = 192.5, y = -2x.$

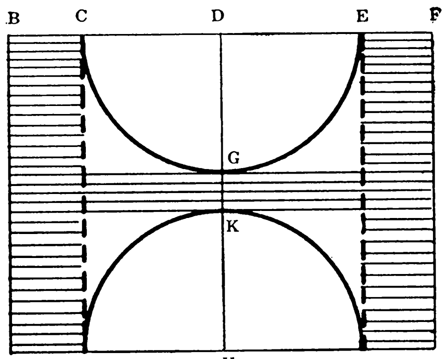

FIG. 14.
$192.5 < x + \infty, y < -2x.$

Also solved by HORACE OLSON, S. E. URNER, PAUL CAPRON, and J. W. BALDWIN.

MECHANICS.

336. Proposed by C. N. SCHMALL, New York City.

An inclined plane, length l, makes an angle $\phi < (\frac{1}{4}\pi)$ with the horizontal plane through its foot. From its foot, a body is projected upward along the plane, with a velocity equal to that of a falling body at the height h, so as to pass over the top and strike the horizontal plane at the maximum distance, x, from the foot of the inclined plane. Show by methods of the calculus that $x = h/(\sin\phi\cos\phi)$, and that the corresponding value of l is $(2h\cot 2\phi)/\cos\phi$.

SOLUTION BY THE PROPOSER.

Let v_1 = velocity at time of projection; v = velocity on reaching top of plane; a = height of inclined plane; x_1 = horizontal distance traversed by body after leaving plane.

Then, by the given conditions, we have

$$v_1^2 = 2gh, \tag{1}$$

$$a = l \sin \phi, \tag{2}$$

$$v^2 = 2g(h - a), \tag{3}$$

$$x = x_1 + a \cot \phi, \tag{4}$$

or,

$$x_1 = x - a \cot \phi. \tag{5}$$

Now taking the top of the inclined plane as the origin, the equation of the path of the body is

$$\left.\begin{aligned} -a &= x_1 \tan \phi - \frac{g}{2} \frac{x_1^2}{v^2 \cos^2 \phi} \\ &= x_1 \tan \phi - \frac{g}{2} \frac{x_1^2}{2g(h-a)\cos^2 \phi} \end{aligned}\right\} \tag{6}$$

by equation (3). Whence,

$$(h - a)(a + x_1 \tan \phi) = \frac{x_1^2}{4 \cos^2 \phi}$$

which, putting $x - a \cot \phi$ for x_1 from (5), becomes

$$(h - a)x \tan \phi = \frac{(x - a \cot \phi)^2}{4 \cos^2 \phi}.$$

Hence,

$$x^2 - 2x(a \cot \phi \cos 2\phi + h \sin 2\phi) + a^2 \cot^2 \phi = 0, \tag{7}$$

where x is to be made a maximum.

Differentiating with respect to a, we get

$$\{x - (a \cot \phi \cos 2\phi + h \sin 2\phi)\} \frac{dx}{da} - x \cot \phi \cos 2\phi + a \cot^2 \phi = 0. \tag{8}$$

Hence,

$$\frac{dx}{da} = 0, \quad \text{if} \quad x = a \frac{\cot \phi}{\cos 2\phi}.$$

Putting this value of x in (7) and reducing, we get

$$a = h \frac{\cos 2\phi}{\cos^2 \phi}. \tag{9}$$

Again, substituting this value of x in the coefficient of dx/da in (8), we get

$$a \frac{\cot \phi}{\cos 2\phi} - a \cot \phi \cos 2\phi - h \sin 2\phi,$$

which is nearly equal to $h \sin 2\phi$.

It is evident, therefore, that as a increases through the value $h(\cos 2\phi)/(\cos^2 \phi)$, the coefficient of dx/da remains about equal to $h \sin 2\phi$, a positive quantity, while the remainder of equation (8), namely, $-x \cot \phi \cos 2\phi + a \cot^2 \phi$, changes from $-$ to $+$, for x is constant and a is increasing. Hence, dx/da changes from $+$ to $-$. Hence x is a maximum when

$$x = a \frac{\cot \phi}{\cos 2\phi} = \frac{h}{\sin \phi \cos \phi}. \qquad \text{by (9)}$$

Also, from (2), we get

$$l = \frac{a}{\sin \phi} = 2h \frac{\cot 2\phi}{\cos \phi}. \qquad \text{by (9)}$$

Also solved by O. S. ADAMS and HORACE OLSON.

NUMBER THEORY.

255. Proposed by FRANK IRWIN, University of California.

Given any arithmetical progression whose first term a and common difference d are relatively prime integers, and any finite set of positive integers $m_1, m_2, \cdots$ also relatively prime to d, it is required to determine an integer n such that the multiples of $m_1, m_2, \cdots$ may occupy the same